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Three important papers were written during the grant period. These were 1. Instability of periodic states for the Sivashinsky equation. 2. On Cahn-Hilliard type equation. 3. Stable patterns in a viscous diffusion equation all the above dealt with solidification and phase change/separation.

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Final Technical Report
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June 26, 1989

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Final Report

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1. Directional Solidification

In the paper, "Instability of periodic steady states for the Sivashinsky equation", it is demonstrated that periodic one dimensional steady states of the equation

$$\begin{aligned} &u_t = (-u + \frac{1}{2}u^2 - k u_{xx})_{xx} - \alpha u \\ &u_x(0) = u_{xxx}(0) = 0 \\ &u_x(L) = u_{xxx}(L) = 0 \end{aligned} \tag{1}$$

if they exist, must be unstable in the sense that the associated eigenvalue problem possesses unstable eigenvalues. Symmetric steady states are likewise shown to be unstable. These results cast serious doubt on the ability of equation (1) to sensibly model the persistence of stable cellular structures in directional solidification.

2. Directional Solidification and Phase Separation

"On Cahn-Hilliard type equations", studies equations in R^n, $n < 4$ of the form

$$\begin{aligned} &u_t = \Delta(f(u) - K\Delta u) \qquad &x \in \Omega \\ &n \cdot \nabla u = n \cdot \nabla \Delta u = 0 \qquad &x \in \partial\Omega \end{aligned} \tag{2}$$

where $|f(x)| \to \infty$ as $x \to \pm\infty$. Equation (2) generalizes both the Cahn-Hilliard equation and the Sivashinsky equation in the small segregation coefficient limit. In terms of nonlinear stability of the planar state, two theorems are given which prescribe conditions under which solutions decay exponentially fast to their mean. In one-dimension it is shown that all nonmonotone steady states are dynamically unstable.

3. Phase Separation

The paper, "Stable patterns in a viscous diffusion equation" (with R.L. Pego) treats the equation

$$\begin{aligned} &u_t = \Delta(f(u) + \nu u_t) \qquad &x \in \Omega \\ &n \cdot \nabla(f(u) + \nu u_t) \qquad &x \in \partial\Omega \end{aligned} \tag{3}$$

where Ω is a bounded domain and where f is assumed to be nonmonotone. Equation (3) arises in the context of phase separation of viscous binary mixtures. Global existence for initial data in L_∞ is proven using invariant regions. Discontinuities in the initial data persist and no new discontinuities may appear in finite time. If f satisfies a certain "nondegeneracy" condition, then solutions converge to steady state. For equation (3) steady states consist of all bounded measureable functions $u_e(x)$ which satisfy $f(u_e(x))$ =constant. If $f'(u) > 0$ a.e., then $u_e(x)$ is dynamically stable. Smooth solutions may converge asymptotically to discontinuous solutions, and solutions may converge to steady states which are not absolute minimizers of the free energy.